Online Appendix

Supplementary Materials for "A Structural Model of Productivity, Uncertain Demand, and Export Dynamics"

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A Robustness of Productivity Measures

In this appendix, I examine the robustness of the productivity measure by comparing productivity estimates from various popular methodologies in literature, including Olley and Pakes (1996), Levinsohn and Petrin (2003), and Wooldridge (2009). The key idea in these methodologies is to use observable variables (such as investment and material inputs) to control for correlation between input levels and the unobserved productivity process. In particular, Olley and Pakes (1996) use investment, conditional on capital stock, in the control function of productivity. Levinsohn and Petrin (2003) show how to use material inputs, rather than investment, in this proxy approach. Wooldridge (2009) demonstrates how to implement these approaches as joint estimation within a two-equation generalized method of moments to gain efficiency of estimators.

In my application, I choose to follow Levinsohn and Petrin (2003) to use material inputs as the proxy because firm-level material expenditure is reported and is non-zero for all firms in all years, while investment is zero or missing for some firms in some years. My objective is to recover a measure of domestic-revenue-based productivity. This is a combination of the physical productivity and domestic demand, which is $\tilde{\omega}$ in equation (4). In the model, the firm makes their input decisions (including material inputs) after observing the domestic demand and physical productivity. Thus, I use material inputs to control for $\tilde{\omega}$ in the estimation of equation (29), a relationship between domestic revenue, $\tilde{\omega}$ and other observed variables. Such methodology of recovering the combination of the physical productivity and domestic demand from the domestic revenue is also adopted in Aw et al. (2011) to control heterogeneous export profitability in their study of export participation and R&D decisions. Alternatively, one could use material inputs in producing domestic sales as the proxy (by weighting the total material inputs by the share of domestic revenue to the total revenue). This approach bears an assumption that the share of inputs used in the domestic production is equal to the share of domestic production in the total production.

The estimates of parameters of marginal cost and the productivity process using different proxy choices from different methodologies are reported in A1. In particular, column (1) and (2) are the estimates following Levinsohn and Petrin (2003) to use material inputs and domestic-used material inputs for the proxy, respectively; column (3) is the estimate following Olley and Pakes (1996) to use investment as the proxy; and column (4) is the estimate following Wooldridge (2009) with material inputs as the proxy.

Overall, the parameter estimates are similar across different methods, and the productivity estimates are also highly correlated (with correlation coefficients ranged from 0.54 to 0.97). Finally, as demonstrated in Table A2, A3, and A4, the reduced form patterns between export participation and productivity using different productivity measures are similar to the one reported in Table 2. This suggests that these different productivity measures play qualitatively similar roles in driving export participation.

Table A1: Robustness: estimates of parameters of marginal cost and productivity evolution

	(1) LP (material)	(2) LP (domestic material)	(3) OP (investment)	(4) Wooldridge (material)
γ_k	-0.053	-0.052	-0.057	-0.050
	(0.006)	(0.002)	(0.001)	(0.003)
g_0	0.0412	0.043	-0.018	-0.065
	(0.008)	(0.003)	(0.003)	(0.003)
g_1	0.872	0.865	0.928	0.917
	(0.005)	(0.005)	(0.003)	(0.006)

Table A2: Regressions of export participation using $\tilde{\omega}$ recovered from LP (proxy: domestic material)

	(1) Linear	(2) Linear	(3) Linear	(4) Probit	(5) Logit
α_e	0.503***	0.517***	0.426***	0.784***	1.080**
	(0.074)	(0.074)	(0.076)	(0.269)	(0.474)
$\alpha_{\tilde{\omega}}$	0.004	0.006	0.035	0.171	0.360
	(0.053)	(0.052)	(0.052)	(0.200)	(0.348)
$\alpha_{\bar{\zeta}}$	0.049***	0.048***	0.041***	0.140***	0.250***
	(0.012)	(0.012)	(0.012)	(0.042)	(0.070)
α_n			0.015***	0.166***	0.358***
			(0.003)	(0.050)	(0.101)
α_N			-0.001*	-0.043*	-0.083*
			(0.001)	(0.026)	(0.043)
α_{N2}				0.001*	0.002*
				(0.000)	(0.001)
Year	No	Yes	Yes	Yes	Yes
Observations	443	443	443	443	443
R^2	0.280	0.286	0.327	0.329	0.337

Standard errors in parentheses.

* $p < .10$, ** $p < .05$, *** $p < .01$.

Table A3: Regressions of export participation using $\tilde{\omega}$ recovered from OP (proxy: investment)

	(1) Linear	(2) Linear	(3) Linear	(4) Probit	(5) Logit
α_e	0.463***	0.478***	0.390***	0.691**	0.930*
	(0.083)	(0.085)	(0.086)	(0.288)	(0.508)
$\alpha_{\tilde{\omega}}$	0.019	0.042	0.016	0.197	0.438
	(0.106)	(0.108)	(0.107)	(0.391)	(0.686)
$\alpha_{\bar{\zeta}}$	0.054***	0.054***	0.044***	0.155***	0.273***
	(0.013)	(0.013)	(0.013)	(0.045)	(0.075)
α_n			0.015***	0.162***	0.351***
			(0.003)	(0.050)	(0.104)
α_N			-0.001*	-0.044*	-0.085**
			(0.001)	(0.026)	(0.043)
α_{N2}				0.001*	0.002*
				(0.000)	(0.001)
Year	No	Yes	Yes	Yes	Yes
Observations	407	407	407	407	407
R^2	0.255	0.262	0.303	0.313	0.321

Standard errors in parentheses.

* $p < .10$, ** $p < .05$, *** $p < .01$.

Table A4: Regressions of export participation using $\tilde{\omega}$ recovered from Wooldridge (proxy: material)

	(1) Linear	(2) Linear	(3) Linear	(4) Probit	(5) Logit
α_e	0.496***	0.510***	0.423***	0.758***	1.042**
	(0.072)	(0.073)	(0.075)	(0.266)	(0.471)
$\alpha_{\tilde{\omega}}$	0.160**	0.161**	0.130*	0.745**	1.290**
	(0.081)	(0.080)	(0.078)	(0.340)	(0.582)
$\alpha_{\bar{\zeta}}$	0.053***	0.052***	0.043***	0.159***	0.279***
	(0.012)	(0.012)	(0.012)	(0.043)	(0.072)
α_n			0.015***	0.170***	0.364***
			(0.003)	(0.050)	(0.103)
α_N			-0.001**	-0.047*	-0.091**
			(0.001)	(0.026)	(0.044)
α_{N2}				0.001*	0.002*
				(0.001)	(0.001)
Year	No	Yes	Yes	Yes	Yes
Observations	443	443	443	443	443
R^2	0.285	0.292	0.330	0.336	0.343

Standard errors in parentheses.

* $p < .10$, ** $p < .05$, *** $p < .01$.

B Robustness Check: Stricter Definitions of Experienced Firms

In the estimation of the dynamic model, I define a firm as an experienced if the firm exported in the initial year of the sample period. In effect, this is to use the export status in the initial year of the observed sample to approximate the experience of a firm in the pre-sample period. It is an economical approach to deal with the problem that export participation before the sample period is not observed. It comes with a risk of mistakenly defining an entrant as an experienced firm. Nonetheless, in this application, such an approximation is appropriate in general because of the high persistence of export participation. In particular, in the sample used in this paper, only 3.6 percent of exporters are fresh entrants in 2001. This suggests that the potential error of defining a firm as an experienced firm according to its export status in the initial year is far from severe.

In order to investigate how appropriate the approximation is, I check how the estimates of the dynamic parameters change with stricter definitions of experienced firms. Specifically, I define the experienced firms as firms that exported in their first *two years* in the sample period. This is in contrast with the definition involving only the first year in the paper. An even stricter definition is to define the firms as experienced firms only if they exported in all their first *three years* in the sample period. I re-estimate the dynamic model using the two stricter definitions as robustness checks.

Table A5 shows that the estimation results in different checks are quantitatively similar. This is consistent with the argument in the paper that potential error of calling all exporting firms in their initial year as experienced firms is minor.

Table A5: Robustness check, by different definitions of experienced firms

Parameter	m_0^p	m_0^e	σ_0^p	σ_0^e	c^f	c^s	δ
Defined by the first year	−3.654	−1.828	0.459	0.297	0.204	2.707	0.880
	(0.059)	(0.027)	(0.009)	(0.006)	(0.073)	(0.086)	(0.038)
Defined by the first two years	−4.004	−2.082	0.447	0.302	0.136	2.504	0.755
	(0.048)	(0.028)	(0.008)	(0.006)	(0.080)	(0.033)	(0.034)
Defined by the first three years	−4.093	−2.168	0.445	0.306	0.085	2.449	0.732
	(0.036)	(0.020)	(0.006)	(0.004)	(0.053)	(0.022)	(0.023)

C Dynamic Estimation Details

I describe the strategy to estimate the dynamic parameters by three stages, as in Rust (1987). The first stage estimates θ_1 via the first partial likelihood. In particular, I estimate θ_1 using the data on the number of transactions and export participation of each firm in each period. To be specific, $\hat{\theta}_1$ is obtained as

$$\hat{\theta}_1 = \arg\max_{\theta_1} \sum_{i,t} \ln(\ell^1(n_{it}; \theta_1)). \tag{1}$$

Note that in this estimation, there is no need to calculate the value function.

Then, with the estimated $\hat{\theta}_1$, the second stage is to estimate θ_2 via the second partial likelihood:

$$\hat{\theta}_2 = \arg\max_{\theta_2} \sum_{i,t} \ln(\ell^2(e_{it}|s_{it}, e_{it-1}; \hat{\theta}_1, \theta_2)). \tag{2}$$

In principle, this stage requires solving the value function at each evaluation of ℓ^2. I use the methodology of Mathematical Program with Equilibrium Constraints (MPEC), which is established by Su and Judd (2012) and Dubé et al. (2012) and further implemented in Barwick and Pathak (2015). Specifically, I cast the Bellman equation as a model constraint in the estimation procedure that has to be satisfied at the parameter estimates. The detailed algorithm is described in D.

The third stage is to use the estimated $(\hat{\theta}_1, \hat{\theta}_2)$ as an initial starting point to produce an efficient estimate of θ via the full likelihood:

$$\hat{\theta} = \arg\max_{\theta} \sum_{i,t} \ln(\ell^f(e_{it}, n_{it}|s_{it}, e_{it-1}; \theta_1, \theta_2)). \tag{3}$$

This stage also involves the internal evaluation of the value function for each evaluation of ℓ^f. This estimation yields a consistent estimator of asymptotic covariance matrix for θ. Nonetheless, the estimate of θ from this stage is usually identical to the estimates from the first two stages (Rust, 1987).

D Value Function Approximation

The typical implementation of the Nested Fixed Point algorithm may lead to inconsistent estimates of the dynamic parameters. Also, the high dimension of state variables in this study also implies significant computational burden in applying the Nested Fixed Point algorithm. In order to address these issues, I use the methodology of Mathematical Program with Equilibrium Constraints in the dynamic estimation. This appendix describes the detailed implementation.

The implementation follows the approach proposed and developed by Su and Judd (2012) and Dubé et al. (2012) and further implemented by Barwick and Pathak (2015). I approximate the unknown value function using Sieves with parametric basis functions and cast the Bellman equation as a model constraint in the estimation procedure that has to be satisfied at the parameter estimates. Throughout this appendix, I suppress the firm index i to simplify the notations.

First, I follow Rust (1987) to define the expected value function as:

$$EV(s_t, e_{t-1}, e_t) \equiv E_{s_{t+1}, e_t, \epsilon}[V(s_{t+1}, e_t, \epsilon)|s_t, e_{t-1}, e_t], \tag{4}$$

where $\epsilon = (\epsilon(0), \epsilon(1))$ is the vector of the trade cost shocks.

Given the Type-I Extreme distribution of ϵ, the Bellman equation (20) can be written as:

$$\begin{aligned} &EV(s_t, e_{t-1}, e_t) \\ &= \sum_{s_{t+1}, e_t} \ln\left\{\sum_{e_{t+1}} \exp[u(s_{t+1}, e_t) + \beta EV(s_{t+1}, e_t, e_{t+1})]\right\} \times \Pr(s_{t+1}, e_t|s_t, e_{t-1}, e_t). \end{aligned} \tag{5}$$

With slight abuse of notation, I choose a set of M polynomial functions $\{f_m(s_t)\}_{m=1}^M$ as basis functions to approximate the unknown value function:

$$EV(s_t, e_{t-1}, e_t) \approx \sum_{m=1}^{M} b_m(e_{t-1}, e_t) f_m(s_t). \tag{6}$$

where $b_m(e_{t-1}, e_t)$ is the unknown coefficient associated with basis function m and export state (e_{t-1}, e_t). That is, I allow for a very flexible function form so that the value function evaluated at different export status (e_{t-1}, e_t) varies.

Plug (6) in to (5), I get:

$$\sum_{m=1}^{M} b_j(e_{t-1}, e_t) f_m(s_t)$$
$$\approx \sum_{s_{t+1}, e_t} \ln \left\{ \sum_{e_{t+1}} \exp[u(s_{t+1}, e_t) + \beta \sum_{m=1}^{M} b_m(e_t, e_{t+1}) f_m(s_{t+1})] \right\} \times \Pr(s_{t+1}, e_t | s_t, e_{t-1}, e_t). \tag{7}$$

Barwick and Pathak (2015) point out that this equation holds approximately at all data states. I follow their approach to choose $\{b_m(e_{t-1}, e_t)\}_{m=1}^{M}$ to fit this nonlinear equation in "least-squared-residuals":

$$\{\hat{b}_m(e_{t-1}, e_t)\}_{m=1}^{M} = \text{argmin}_{\{b_m(e_{t-1}, e_t)\}_{m=1}^{M}} \Bigg\| \sum_{m=1}^{M} b_m(e_{t-1}, e_t) f_m(s_t) -$$
$$\sum_{s_{t+1}, e_t} \ln \left\{ \sum_{e_{t+1}} \exp[u(s_{t+1}, e_t) + \beta \sum_{m=1}^{M} b_m(e_t, e_{t+1}) f_m(s_{t+1})] \right\} \times \Pr(s_{t+1}, e_t | s_t, e_{t-1}, e_t) \Bigg\|_2 . \tag{8}$$

The conditional choice probability function (36) can be re-written as:

$$\begin{aligned} & \ell^2(e_{it} | s_t, e_{t-1}; \hat{\theta}_1, \theta_2) \\ = & \Pr(e_t = 1 | s_t, e_{t-1}; \theta_1, \theta_2) \\ = & \frac{\exp[u(s_{it}, e_{it-1}) + \beta \sum_{m=1}^{M} b_j(e_t, 1) f_j(s_{t+1})]}{\exp[u(s_{it}, e_{it-1}) + \beta \sum_{m=1}^{M} b_j(e_t, 1) f_j(s_{t+1})] + \exp[\beta \sum_{m=1}^{M} b_j(e_t, 0) f_j(s_{t+1})]}, \end{aligned} \tag{9}$$

Finally, the estimation of θ_2 via the Maximum Likelihood method (2) can be cast with the value function approximation as a constraint:

$$\begin{aligned} \hat{\theta}_2 = \arg\max_{\theta_2} \sum_{i,t} \ln(\ell^2(e_{it} | s_t, e_{t-1}; \hat{\theta}_1, \theta_2)) \\ \text{subject to} \quad (8). \end{aligned} \tag{10}$$